
REVIEW 2

1. Compute the derivative of the following functions :

i) $f(x) = \ln(3x^2 + 1) + \arctan(x)$

ii) $f(x) = \frac{\cos(x^2)}{\log_3(x)}$

2. Compute the following limits :

i) $\lim_{x \rightarrow 0} \frac{e^{3x} - 1 - 3x}{x^2}$

ii) $\lim_{x \rightarrow +\infty} x^2 \ln \left(1 + \frac{1}{2x^2}\right)$

3. A box with an open top has vertical sides, a square bottom and a volume of 8 cubic meters. Given that the box has the least possible surface area, find its dimension.

4. If $f(1) = 1$ and $f'(x) = e^x f(x)$, find $f''(1)$.

5. Show that the function $f(x) = 7 \ln(x) - 17x + 6x^2 - \frac{2}{3}x^3$ has a critical point at $x = 1$. Is it a local maximum, a local minimum or neither?

6. Find the absolute maximum and minimum of the function $f(x) = 3 + 2 \sin(x) + \cos^2(x) - \sin^2(x)$ on the interval $[0, \pi]$.

7. Consider the function $f(x) = \frac{x}{\ln(x)}$.

- i) Determine for which values of x the function is well-defined.
- ii) Find and classify the critical points of $f(x)$.
- iii) Determine the intervals where the function is increasing.
- iv) Find the inflection points of $f(x)$.
- v) Determine the intervals where the function is concave-up.
- vi) Compute $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow 0^+} f(x)$.
- vii) Compute $\lim_{x \rightarrow 1^+} f(x)$ and $\lim_{x \rightarrow 1^-} f(x)$.
- viii) Sketch the graph of $f(x)$.